**VII. Complex Sequences & Series:**

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| **Limit Laws** (because series so we only have ) | | | | |
| **Basic Rule** |  |  | |  |
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| **If and are continuous functions and is a constant:** | | | |
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| **Squeeze Theorem** |  | | | |
| **L’ Hospital Rule** |  | | | |
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| **Complex Sequence** | * A complex sequence is a list of complex numbers , called terms * Notation: * The complex sequence with converges when: * The radius of convergence: |
| **Complex Series** | * Given a complex sequence , the complex series or is: * The partial sum of the series is: * A series is convergent when: then * A series is divergent when: not exist |
| **Geometric Series** | * The geometric series has the form of * Converges: & sum is * Diverges: |

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| **Real & Imaginary Parts Test** | * A series converges when:   converges & has the sum  converges & has the sum  The sum of the series: |
| **Divergent Test** | * A series diverges when  **doesn’t exist** or |
| **Absolute Convergent** | * A series has the absolute series : * If **converges** ⇒ **is absolute converges** ⇒  **converges** |
| **Comparison Test** | * If we have a series and a series (with nonnegative terms):   + If  **converges** &⇒ absolute converges   + If  **diverges** &⇒diverges * Note: we should choose = Geometric Series or P-Series * Note: chọn x có mũ lớn nhất của tử và mẫu |
| **Ratio Test** | * If we have a series :   + If ⟹ absolute converges   + If or ⟹ diverges   + If ⟹ can’t conclude |
| **Root Test** | * If we have a series :   + If ⟹ absolute converges   + If or ⟹ diverges   + If ⟹ can’t conclude |

**VIII. Power Series**

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| **Definition** | * A power series in powers of :   + is the coefficients (complex or real constant)   + is the center of the series (complex or real constant) * A power series in powers of : | |
| **Radius of Convergent** | * The power series converges when: * The power series diverges when: |  |
| * Calculate the radius of convergence R:   + If ⇒ power series converges when   + If ⇒ power series converges for all   + If ⇒ power series converges only at | |
| Example | Finding the power series (manual way) of a complex function:   * Step 1: check the regions   + If then the power series will have the form   + If then the power series will have form   + If then the power series will have form * Step 2: change the function according to the form:   + If ⇒ change into   + If ⇒ change into   + If ⇒ change into * Step 3: using the binomial series expansion | |
| **Find the Power series of**   1. **In the region** 2. **In the region** 3. **In the region** 4. (cho nằm ở tử):  * Apply the binomial series expansion:  1. (cho nằm ở mẫu):  * Apply the binomial series expansion: * Apply the binomial series expansion:   **Find the Power series of**   1. **In the region** 2. **In the region** 3. **In the region** 4. :  * Apply the binomial series expansion:  1. :  * Apply the binomial series expansion. We have: * Apply the binomial series expansion. We have: | |
| Example | 🡪 Find the radius and center of convergent?   * We have: * So the radius of convergence is and the center is   🡪 Find the radius and center of convergent?   * We have: * So the radius of convergence is and the center is   🡪 Find the radius and center of convergent?   * We have: * So the radius of convergence is and the center is   🡪 Find the radius and center of convergent?   * We change the series to the right power series form: * We have: * So the radius of convergence is and the center is | |

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| **Power Series as Function** | If the power series has radius , then the function is defined:   * The derivative of : * The anti-derivative of : |
| Example | Find the Power series of ?   * Using the geometric series (bên dưới) we have:   Find the Power series of ?   * We have: * So: * We have: * Therefore: |

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| **Taylor Series** | * The Taylor series expansion of a complex function about : |
| Maclaurin Series | * When the Taylor Series 🡪 the Maclaurin Series: |
| Example | Find the Taylor series of about up to term ?   * First, we should break the function into fractions * We have:  |  |  |  | | --- | --- | --- | | n |  |  | | 0 |  |  | | 1 |  |  | | 2 |  |  | | 3 |  |  | | 4 |  |  |  * The Taylor series expansion is:   Find the Taylor series of about up to first 4 term?   * We have:  |  |  |  | | --- | --- | --- | | n |  |  | | 0 |  |  | | 1 |  |  | | 2 |  |  | | 3 |  |  |  * The Taylor series expansion is:   Find the Taylor series of about up to first 4 term?   * We have:  |  |  |  | | --- | --- | --- | | n |  |  | | 0 |  |  | | 1 |  |  | | 2 |  |  | | 3 |  |  | | 4 |  |  | | 5 |  | 0 |  * The Taylor series expansion is: |

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| Special Taylor Series | |  |  | | --- | --- | | Geometric Series |  | | Exponential Function |  | |  | | Trigonometric Function |  | | Hyperbolic Function |  | | Logarithm |  | |
| Practical Method to find Taylor Series | **Substitution**  Find the Maclaurin series of ?  Find the Maclaurin series of ?  Find the Maclaurin series of ?  Find the Maclaurin series of ?  Find the Maclaurin series of ?   * We have: * So: |
| **Integration – Differentiation**  Find the Maclaurin series of ?  Find the Maclaurin series of ?   * We have: * And: * So:   Find the Maclaurin series of ?   * We have: * So: * Therefore: |
| Find the Maclaurin series of ?   * We have: * So:   Find the Maclaurin series of ?   * We have: * So:   Find the Maclaurin series of ?   * We have: * So:   Find the Maclaurin series of ?   * We have: * So: |
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